A*rticle*

Intuitionistic fuzzy rough sets: at the crossroads of imperfect knowledge

Chris Cornelis, Martine De Cock and Etienne E. Kerre

Department of Applied Mathematics and Computer Science, Ghent University, Fuzziness and Uncertainty Modelling Research Unit, Krijgslaan 281 (S9), B-9000 Ghent, Belgium
E-mail: {chris.cornelis, martine.decock, etienne.kerre}@rug.ac.be

Abstract: *Just like rough set theory, fuzzy set theory addresses the topic of dealing with imperfect knowledge. Recent investigations have shown how both theories can be combined into a more flexible, more expressive framework for modelling and processing incomplete information in information systems. At the same time, intuitionistic fuzzy sets have been proposed as an attractive extension of fuzzy sets, enriching the latter with extra features to represent uncertainty (on top of vagueness). Unfortunately, the various tentative definitions of the concept of an 'intuitionistic fuzzy rough set' that were raised in their wake are a far cry from the original objectives of rough set theory. We intend to fill an obvious gap by introducing a new definition of intuitionistic fuzzy rough sets, as the most natural generalization of Pawlak's original concept of rough sets.*

Keywords: rough set theory, intuitionistic fuzzy set theory, *L*-fuzzy set theory, incomplete information, lower and upper approximation

1. Introduction

As a new trend in the attempts to combine the best of several worlds, very recently all kinds of suggestions for approaches merging rough set theory and intuitionistic fuzzy set theory have started to appear. The present

evolution vividly reminds us of the origin of fuzzy rough set theory, as the (so far) happy marriage of fuzzy set theory and rough set theory. A remarkable difference, however, is that in the latter case a long engagement period with intense discussions concerning the relation between fuzzy set theory and rough set theory preceded the marriage, (and in a way is still going on, simultaneously with the research on the new hybrid theory). So far, this comparison stage seems very limited for the combination of intuitionistic fuzzy set theory and (fuzzy) rough set theory. As far as we know, only Çoker (1998) went into the matter by claiming that fuzzy rough sets are intuitionistic fuzzy sets (Chakrabarty *et al.*, 1998; Samanta & Mondal, 2001; Jena & Ghosh, 2002; Rizvi *et al.*, 2002), which appears to be shattering the dream of a new hybrid theory.

On the other hand, there exist many views on the notion 'rough set' which can be grouped into two main streams. Several suggested options for fuzzification have led to an even greater number of views on the notion 'fuzzy rough set'. Typically, under the same formal umbrella, they can be further generalized to the notion '*L*-fuzzy rough set' where the membership degrees are taken from some suitable lattice *L* which is not necessarily the unit interval. On top of this, there exist semantically different interpretations of intuitionistic fuzzy set theory (which is a special kind of *L*-fuzzy set theory). Needless to say, when trying to compare and/or to combine rough set theory, fuzzy set theory and intuitionistic fuzzy set theory, one finds oneself at a complicated crossroads with an abundance of possible ways to proceed. The aim of this paper is to provide the reader with a road map. We do this by mapping out research results obtained so far in the literature, as well as by exploring by ourselves a very important road which was virgin territory until now.

However, before we can prepare a hybrid theory, it is absolutely necessary to check the origin of all ingredients, for they can have an important influence on the flavour of the resulting product! For this reason we start the paper with a short overview of all set theoretical models involved (Section 2). Making such a study forces us into thinking about relationships between them. Staying within the scope of the paper, we will only focus on intuitionistic fuzzy set theory versus fuzzy rough set theory (Section 3). After a critical examination of the added value of a hybrid intuitionistic fuzzy rough set theory, in Section 4 we present an overview of existing approaches (all originated independently from the others). Finally we fill an obvious gap by a very natural generalization of Pawlak's original concept of a rough set.